Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise A, Question 1

Question:

Simplify these fractions:

(a) $\frac{4x^4 + 5x^2 - 7x}{x}$

(b) $\frac{7x^8 - 5x^5 + 9x^3 + x^2}{x}$

(c) $\frac{-2x^3 + x}{x}$

(d) $\frac{-x^4 + 4x^2 + 6}{x}$

(e) $\frac{7x^5 - x^3 - 4}{x}$

(f) $\frac{8x^4 - 4x^3 + 6x}{2x}$

(g) $\frac{9x^2 - 12x^3 - 3x}{3x}$

(h) $\frac{8x^5 - 2x^3}{4x}$

(i) $\frac{7x^3 - x^4 - 2}{5x}$

(j) $\frac{-4x^2 + 6x^4 - 2x}{-2x}$

(k) $\frac{-x^8 + 9x^4 + 6}{-2x}$

(l) $\frac{-9x^9 - 6x^4 - 2}{-3x}$

Solution:

(a) $\frac{4x^4 + 5x^2 - 7x}{x} = \frac{4x^4}{x} + \frac{5x^2}{x} - \frac{7x}{x} = 4x^3 + 5x - 7$

(b) $\frac{7x^8 - 5x^5 + 9x^3 + x^2}{x} = \frac{7x^8}{x} - \frac{5x^5}{x} + \frac{9x^3}{x} + \frac{x^2}{x} = 7x^7 - 5x^4 + 9x^2 + x$

(c) $\frac{-2x^3 + x}{x} = \frac{-2x^3}{x} + \frac{x}{x} = -2x^2 + 1$

(d) $\frac{-x^4 + 4x^2 + 6}{x} = \frac{-x^4}{x} + \frac{4x^2}{x} + \frac{6}{x} = -x^3 + 4x + \frac{6}{x}$

(e) $\frac{7x^5 - x^3 - 4}{x} = \frac{7x^5}{x} - \frac{x^3}{x} - \frac{4}{x} = 7x^4 - x^2 - \frac{4}{x}$

(f) $\frac{8x^4 - 4x^3 + 6x}{2x} = \frac{8x^4}{2x} - \frac{4x^3}{2x} + \frac{6x}{2x} = 4x^3 - 2x^2 + 3$

(g) $\frac{9x^2 - 12x^3 - 3x}{3x} = \frac{9x^2}{3x} - \frac{12x^3}{3x} - \frac{3x}{3x} = 3x - 4x^2 - 1$

(h) $\frac{8x^5 - 2x^3}{4x} = \frac{8x^5}{4x} - \frac{2x^3}{4x} = 2x^4 - \frac{x^2}{2}$

(i) $\frac{7x^3 - x^4 - 2}{5x} = \frac{7x^3}{5x} - \frac{x^4}{5x} - \frac{2}{5x} = \frac{7x^2}{5} - \frac{x^3}{5} - \frac{2}{5x}$

(j) $\frac{-4x^2 + 6x^4 - 2x}{-2x} = \frac{-4x^2}{-2x} + \frac{6x^4}{-2x} - \frac{2x}{-2x}$

$= \frac{2x^2}{x} - \frac{3x^4}{x} + 1$

$= 2x - 3x^3 + 1$

(k) $\frac{-x^8 + 9x^4 + 6}{-2x} = \frac{-x^8}{-2x} + \frac{9x^4}{-2x} + \frac{6}{-2x}$

$= \frac{x^8}{2x} - \frac{9x^4}{2x} - \frac{3}{x}$

$= \frac{x^7}{2} - \frac{9x^3}{2} - \frac{3}{x}$

(l) $\frac{-9x^9 - 6x^4 - 2}{-3x} = \frac{-9x^9}{-3x} - \frac{6x^4}{-3x} - \frac{2}{-3x}$

$= \frac{3x^9}{x} + \frac{2x^4}{x} + \frac{2}{3x}$

$= 3x^8 + 2x^3 + \frac{2}{3x}$

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Exercise A, Question 2

Question:

Simplify these fractions as far as possible:

(a) $\frac{(x+3)(x-2)}{(x-2)}$

(b) $\frac{(x+4)(3x-1)}{(3x-1)}$

(c) $\frac{(x+3)^2}{(x+3)}$

(d) $\frac{x^2+10x+21}{(x+3)}$

(e) $\frac{x^2+9x+20}{(x+4)}$

(f) $\frac{x^2+x-12}{(x-3)}$

(g) $\frac{x^2+x-20}{x^2+2x-15}$

(h) $\frac{x^2+3x+2}{x^2+5x+4}$

(i) $\frac{x^2+x-12}{x^2-9x+18}$

(j) $\frac{2x^2+7x+6}{(x-5)(x+2)}$

(k) $\frac{2x^2+9x-18}{(x+6)(x+1)}$

(l) $\frac{3x^2-7x+2}{(3x-1)(x+2)}$

(m) $\frac{2x^2+3x+1}{x^2-x-2}$

(n) $\frac{x^2 + 6x + 8}{3x^2 + 7x + 2}$

(o) $\frac{2x^2 - 5x - 3}{2x^2 - 9x + 9}$

Solution:

(a) $\frac{(x + 3)(x - 2)}{(x - 2)}$

$= \frac{(x+3)(x-2)}{(x-2)}$

$= x + 3$

(b) $\frac{(x + 4)(3x - 1)}{(3x - 1)}$

$= \frac{(x+4)(3x-1)}{(3x-1)}$

$= x + 4$

(c) $\frac{(x + 3)^2}{(x + 3)}$

$= \frac{(x+3)(x-3)}{(x-3)}$

$= x + 3$

(d) $\frac{x^2 + 10x + 21}{x + 3}$

$= \frac{(x+7)(x+3)}{(x+3)}$

$= x + 7$

(e) $\frac{x^2 + 9x + 20}{x + 4}$

$= \frac{(x+4)(x+5)}{(x+4)}$

$= x + 5$

(f) $\frac{x^2 + x - 12}{x - 3}$

$= \dfrac{(x-3)(x+4)}{(x-3)}$

$= x + 4$

(g) $\dfrac{x^2 + x - 20}{x^2 + 2x - 15}$

$= \dfrac{(x+5)(x-4)}{(x+5)(x-3)}$

$= \dfrac{x - 4}{x - 3}$

(h) $\dfrac{x^2 + 3x + 2}{x^2 + 5x + 4}$

$= \dfrac{(x+2)(x+1)}{(x+4)(x+1)}$

$= \dfrac{x + 2}{x + 4}$

(i) $\dfrac{x^2 + x - 12}{x^2 - 9x + 18}$

$= \dfrac{(x+4)(x-3)}{(x-6)(x-3)}$

$= \dfrac{x + 4}{x - 6}$

(j) $\dfrac{2x^2 + 7x + 6}{(x - 5)(x + 2)}$

$= \dfrac{(2x+3)(x+2)}{(x-5)(x+2)}$

$= \dfrac{2x + 3}{x - 5}$

(k) $\dfrac{2x^2 + 9x - 18}{(x + 6)(x + 1)}$

$= \dfrac{(2x-3)(x+6)}{(x+6)(x+1)}$

$= \dfrac{2x - 3}{x + 1}$

(l) $\frac{3x^2 - 7x + 2}{(3x - 1)(x + 2)}$

$= \frac{(3x-1)(x-2)}{(3x-1)(x+2)}$

$= \frac{x - 2}{x + 2}$

(m) $\frac{2x^2 + 3x + 1}{x^2 - x - 2}$

$= \frac{(2x+1)(x+1)}{(x-2)(x+1)}$

$= \frac{2x + 1}{x - 2}$

(n) $\frac{x^2 + 6x + 8}{3x^2 + 7x + 2}$

$= \frac{(x+4)(x+2)}{(3x+1)(x+2)}$

$= \frac{x + 4}{3x + 1}$

(o) $\frac{2x^2 - 5x - 3}{2x^2 - 9x + 9}$

$= \frac{(2x+1)(x-3)}{(2x-3)(x-3)}$

$= \frac{2x + 1}{2x - 3}$

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Exercise B, Question 1

Question:

Divide:

(a) $x^3 + 6x^2 + 8x + 3$ by $(x + 1)$

(b) $x^3 + 10x^2 + 25x + 4$ by $(x + 4)$

(c) $x^3 + 7x^2 - 3x - 54$ by $(x + 6)$

(d) $x^3 + 9x^2 + 18x - 10$ by $(x + 5)$

(e) $x^3 - x^2 + x + 14$ by $(x + 2)$

(f) $x^3 + x^2 - 7x - 15$ by $(x - 3)$

(g) $x^3 - 5x^2 + 8x - 4$ by $(x - 2)$

(h) $x^3 - 3x^2 + 8x - 6$ by $(x - 1)$

(i) $x^3 - 8x^2 + 13x + 10$ by $(x - 5)$

(j) $x^3 - 5x^2 - 6x - 56$ by $(x - 7)$

Solution:

(a)

$$
\begin{array}{r|l}
x + 1 & x^2 + 5x + 3 \\
\cline{2-2}
 & x^3 + 6x^2 + 8x + 3 \\
 & x^3 + x^2 \\
 & \quad 5x^2 + 8x \\
 & \quad 5x^2 + 5x \\
 & \qquad 3x + 3 \\
 & \qquad 3x + 3 \\
 & \quad 0
\end{array}
$$

Answer is $x^2 + 5x + 3$

(b)

$$x + 4 \overline{)\, x^3 + 10x^2 + 25x + 4}$$

$x^2 + 6x + 1$

$x^3 + 4x^2$

$6x^2 + 25x$

$6x^2 + 24x$

$x + 4$

$x + 4$

0

Answer is $x^2 + 6x + 1$

(c)

$$x + 6 \overline{)\, x^3 + 7x^2 - 3x - 54}$$

$x^2 + x - 9$

$x^3 + 6x^2$

$x^2 - 3x$

$x^2 + 6x$

$-9x - 54$

$-9x - 54$

0

Answer is $x^2 + x - 9$

(d)

$$x + 5 \overline{)\, x^3 + 9x^2 + 18x - 10}$$

$x^2 + 4x - 2$

$x^3 + 5x^2$

$4x^2 + 18x$

$4x^2 + 20x$

$-2x - 10$

$-2x - 10$

0

Answer is $x^2 + 4x - 2$

(e)

$$x + 2 \overline{)\, x^3 - x^2 + x + 14}$$

$x^2 - 3x + 7$

$x^3 + 2x^2$

$-3x^2 + x$

$-3x^2 - 6x$

$7x + 14$

$7x + 14$

0

Answer is $x^2 - 3x + 7$

(f)

$$x - 3 \overline{)\; x^3 + x^2 - 7x - 15}$$

$$x^2 + 4x + 5$$

$$x^3 - 3x^2$$

$$4x^2 - 7x$$

$$4x^2 - 12x$$

$$5x - 15$$

$$5x - 15$$

$$0$$

Answer is $x^2 + 4x + 5$

(g)

$$x - 2 \overline{)\; x^3 - 5x^2 + 8x - 4}$$

$$x^2 - 3x + 2$$

$$x^3 - 2x^2$$

$$-3x^2 + 8x$$

$$-3x^2 + 6x$$

$$2x - 4$$

$$2x - 4$$

$$0$$

Answer is $x^2 - 3x + 2$

(h)

$$x - 1 \overline{)\; x^3 - 3x^2 + 8x - 6}$$

$$x^2 - 2x + 6$$

$$x^3 - x^2$$

$$-2x^2 + 8x$$

$$-2x^2 + 2x$$

$$6x - 6$$

$$6x - 6$$

$$0$$

Answer is $x^2 - 2x + 6$

(i)

$$x - 5 \overline{)\; x^3 - 8x^2 + 13x + 10}$$

$$x^2 - 3x - 2$$

$$x^3 - 5x^2$$

$$-3x^2 + 13x$$

$$-3x^2 + 15x$$

$$-2x + 10$$

$$-2x + 10$$

$$0$$

Answer is $x^2 - 3x - 2$

(j)

$$
\begin{array}{rl}
 & x^2 + 2x + 8 \\
x - 7 \;\big)\; & x^3 - 5x^2 - 6x - 56 \\
 & x^3 - 7x^2 \\
 & \quad 2x^2 - 6x \\
 & \quad 2x^2 - 14x \\
 & \qquad 8x - 56 \\
 & \qquad 8x - 56 \\
 & \quad 0
\end{array}
$$

Answer is $x^2 + 2x + 8$

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Exercise B, Question 2

Question:

Divide:

(a) $6x^3 + 27x^2 + 14x + 8$ by $(x + 4)$

(b) $4x^3 + 9x^2 - 3x - 10$ by $(x + 2)$

(c) $3x^3 - 10x^2 - 10x + 8$ by $(x - 4)$

(d) $3x^3 - 5x^2 - 4x - 24$ by $(x - 3)$

(e) $2x^3 + 4x^2 - 9x - 9$ by $(x + 3)$

(f) $2x^3 - 15x^2 + 14x + 24$ by $(x - 6)$

(g) $-3x^3 + 2x^2 - 2x - 7$ by $(x + 1)$

(h) $-2x^3 + 5x^2 + 17x - 20$ by $(x - 4)$

(i) $-5x^3 - 27x^2 + 23x + 30$ by $(x + 6)$

(j) $-4x^3 + 9x^2 - 3x + 2$ by $(x - 2)$

Solution:

(a)

$$
\begin{array}{r|l}
x + 4 & 6x^2 + 3x + 2 \\
\cline{2-2}
 & 6x^3 + 27x^2 + 14x + 8 \\
 & 6x^3 + 24x^2 \\
 & \quad 3x^2 + 14x \\
 & \quad 3x^2 + 12x \\
 & \quad\quad 2x + 8 \\
 & \quad\quad 2x + 8 \\
 & \quad\quad 0
\end{array}
$$

Answer is $6x^2 + 3x + 2$

(b)

$$
\begin{array}{r|l}
x + 2 & 4x^2 + x - 5 \\
\cline{2-2}
 & 4x^3 + 9x^2 - 3x - 10 \\
 & 4x^3 + 8x^2 \\
 & \quad x^2 - 3x \\
 & \quad x^2 + 2x \\
 & \quad\quad -5x - 10 \\
 & \quad\quad -5x - 10 \\
 & \quad 0
\end{array}
$$

Answer is $4x^2 + x - 5$

(c)

$$
\begin{array}{r|l}
x - 4 & 3x^2 + 2x - 2 \\
\cline{2-2}
 & 3x^3 - 10x^2 - 10x + 8 \\
 & 3x^3 - 12x^2 \\
 & \quad 2x^2 - 10x \\
 & \quad 2x^2 - 8x \\
 & \quad\quad -2x + 8 \\
 & \quad\quad -2x + 8 \\
 & \quad 0
\end{array}
$$

Answer is $3x^2 + 2x - 2$

(d)

$$
\begin{array}{r|l}
x - 3 & 3x^2 + 4x + 8 \\
\cline{2-2}
 & 3x^3 - 5x^2 - 4x - 24 \\
 & 3x^3 - 9x^2 \\
 & \quad 4x^2 - 4x \\
 & \quad 4x^2 - 12x \\
 & \quad\quad 8x - 24 \\
 & \quad\quad 8x - 24 \\
 & \quad 0
\end{array}
$$

Answer is $3x^2 + 4x + 8$

(e)

$$
\begin{array}{r|l}
x + 3 & 2x^2 - 2x - 3 \\
\cline{2-2}
 & 2x^3 + 4x^2 - 9x - 9 \\
 & 2x^3 + 6x^2 \\
 & \quad -2x^2 - 9x \\
 & \quad -2x^2 - 6x \\
 & \quad\quad -3x - 9 \\
 & \quad\quad -3x - 9 \\
 & \quad 0
\end{array}
$$

Answer is $2x^2 - 2x - 3$

(f)

$$\begin{array}{r|l} & 2x^2 - 3x - 4 \\ \hline x - 6 & 2x^3 - 15x^2 + 14x + 24 \\ & 2x^3 - 12x^2 \\ & \quad - 3x^2 + 14x \\ & \quad - 3x^2 + 18x \\ & \qquad - 4x + 24 \\ & \qquad - 4x + 24 \\ & \quad 0 \end{array}$$

Answer is $2x^2 - 3x - 4$

(g)

$$\begin{array}{r|l} & -3x^2 + 5x - 7 \\ \hline x + 1 & -3x^3 + 2x^2 - 2x - 7 \\ & -3x^3 - 3x^2 \\ & \quad 5x^2 - 2x \\ & \quad 5x^2 + 5x \\ & \qquad - 7x - 7 \\ & \qquad - 7x - 7 \\ & \quad 0 \end{array}$$

Answer is $-3x^2 + 5x - 7$

(h)

$$\begin{array}{r|l} & -2x^2 - 3x + 5 \\ \hline x - 4 & -2x^3 + 5x^2 + 17x - 20 \\ & -2x^3 + 8x^2 \\ & \quad - 3x^2 + 17x \\ & \quad - 3x^2 + 12x \\ & \qquad 5x - 20 \\ & \qquad 5x - 20 \\ & \quad 0 \end{array}$$

Answer is $-2x^2 - 3x + 5$

(i)

$$\begin{array}{r|l} & -5x^2 + 3x + 5 \\ \hline x + 6 & -5x^3 - 27x^2 + 23x + 30 \\ & -5x^3 - 30x^2 \\ & \quad 3x^2 + 23x \\ & \quad 3x^2 + 18x \\ & \qquad 5x + 30 \\ & \qquad 5x + 30 \\ & \quad 0 \end{array}$$

Answer is $-5x^2 + 3x + 5$

(j)

$$
\begin{array}{rl}
 & -4x^2 + x - 1 \\
x - 2 \;\big| & -4x^3 + 9x^2 - 3x + 2 \\
 & -4x^3 + 8x^2 \\
 & \quad x^2 - 3x \\
 & \quad x^2 - 2x \\
 & \qquad -x + 2 \\
 & \qquad -x + 2 \\
 & \quad 0
\end{array}
$$

Answer is $-4x^2 + x - 1$

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Exercise B, Question 3

Question:

Divide:

(a) $x^4 + 5x^3 + 2x^2 - 7x + 2$ by $(x + 2)$

(b) $x^4 + 11x^3 + 25x^2 - 29x - 20$ by $(x + 5)$

(c) $4x^4 + 14x^3 + 3x^2 - 14x - 15$ by $(x + 3)$

(d) $3x^4 - 7x^3 - 23x^2 + 14x - 8$ by $(x - 4)$

(e) $-3x^4 + 9x^3 - 10x^2 + x + 14$ by $(x - 2)$

(f) $3x^5 + 17x^4 + 2x^3 - 38x^2 + 5x - 25$ by $(x + 5)$

(g) $6x^5 - 19x^4 + x^3 + x^2 + 13x + 6$ by $(x - 3)$

(h) $-5x^5 + 7x^4 + 2x^3 - 7x^2 + 10x - 7$ by $(x - 1)$

(i) $2x^6 - 11x^5 + 14x^4 - 16x^3 + 36x^2 - 10x - 24$ by $(x - 4)$

(j) $-x^6 + 4x^5 - 4x^4 + 4x^3 - 5x^2 + 7x - 3$ by $(x - 3)$

Solution:

(a)

$$
\begin{array}{r|l}
x + 2 & x^3 + 3x^2 - 4x + 1 \\
\cline{2-2}
 & x^4 + 5x^3 + 2x^2 - 7x + 2 \\
 & x^4 + 2x^3 \\
 & \quad 3x^3 + 2x^2 \\
 & \quad 3x^3 + 6x^2 \\
 & \quad\quad -4x^2 - 7x \\
 & \quad\quad -4x^2 - 8x \\
 & \quad\quad\quad x + 2 \\
 & \quad\quad\quad x + 2 \\
 & \quad\quad\quad 0
\end{array}
$$

Answer is $x^3 + 3x^2 - 4x + 1$

(b)

$$
\begin{array}{r|l}
 & x^3 + 6x^2 - 5x - 4 \\
\hline
x + 5 & x^4 + 11x^3 + 25x^2 - 29x - 20 \\
 & x^4 + 5x^3 \\
 & \quad 6x^3 + 25x^2 \\
 & \quad 6x^3 + 30x^2 \\
 & \quad\quad - 5x^2 - 29x \\
 & \quad\quad - 5x^2 - 25x \\
 & \quad\quad\quad - 4x - 20 \\
 & \quad\quad\quad - 4x - 20 \\
 & \quad 0
\end{array}
$$

Answer is $x^3 + 6x^2 - 5x - 4$

(c)

$$
\begin{array}{r|l}
 & 4x^3 + 2x^2 - 3x - 5 \\
\hline
x + 3 & 4x^4 + 14x^3 + 3x^2 - 14x - 15 \\
 & 4x^4 + 12x^3 \\
 & \quad 2x^3 + 3x^2 \\
 & \quad 2x^3 + 6x^2 \\
 & \quad\quad - 3x^2 - 14x \\
 & \quad\quad - 3x^2 - 9x \\
 & \quad\quad\quad - 5x - 15 \\
 & \quad\quad\quad - 5x - 15 \\
 & \quad 0
\end{array}
$$

Answer is $4x^3 + 2x^2 - 3x - 5$

(d)

$$
\begin{array}{r|l}
 & 3x^3 + 5x^2 - 3x + 2 \\
\hline
x - 4 & 3x^4 - 7x^3 - 23x^2 + 14x - 8 \\
 & 3x^4 - 12x^3 \\
 & \quad 5x^3 - 23x^2 \\
 & \quad 5x^3 - 20x^2 \\
 & \quad\quad - 3x^2 + 14x \\
 & \quad\quad - 3x^2 + 12x \\
 & \quad\quad\quad 2x - 8 \\
 & \quad\quad\quad 2x - 8 \\
 & \quad 0
\end{array}
$$

Answer is $3x^3 + 5x^2 - 3x + 2$

(e)

$$
\begin{array}{r}
-3x^3+3x^2-4x-7 \\
x-2\overline{)\,-3x^4+9x^3-10x^2+x+14} \\
-3x^4+6x^3 \\
\hline
3x^3-10x^2 \\
3x^3-6x^2 \\
\hline
-4x^2+x \\
-4x^2+8x \\
\hline
-7x+14 \\
-7x+14 \\
\hline
0
\end{array}
$$

Answer is $-3x^3+3x^2-4x-7$

(f)

$$
\begin{array}{r}
3x^4+2x^3-8x^2+2x-5 \\
x+5\overline{)\,3x^5+17x^4+2x^3-38x^2+5x-25} \\
3x^5+15x^4 \\
\hline
2x^4+2x^3 \\
2x^4+10x^3 \\
\hline
-8x^3-38x^2 \\
-8x^3-40x^2 \\
\hline
2x^2+5x \\
2x^2+10x \\
\hline
-5x-25 \\
-5x-25 \\
\hline
0
\end{array}
$$

Answer is $3x^4+2x^3-8x^2+2x-5$

(g)

$$
\begin{array}{r}
6x^4-x^3-2x^2-5x-2 \\
x-3\overline{)\,6x^5-19x^4+x^3+x^2+13x+6} \\
6x^5-18x^4 \\
\hline
-x^4+x^3 \\
-x^4+3x^3 \\
\hline
-2x^3+x^2 \\
-2x^3+6x^2 \\
\hline
-5x^2+13x \\
-5x^2+15x \\
\hline
-2x+6 \\
-2x+6 \\
\hline
0
\end{array}
$$

Answer is $6x^4 - x^3 - 2x^2 - 5x - 2$

(h)

$$
\begin{array}{r}
 & -5x^4 + 2x^3 + 4x^2 - 3x + 7 \\
x - 1 \;\overline{\big)\; -5x^5 + 7x^4 + 2x^3 - 7x^2 + 10x - 7} \\
 & -5x^5 + 5x^4 \\
 & \quad 2x^4 + 2x^3 \\
 & \quad 2x^4 - 2x^3 \\
 & \qquad 4x^3 - 7x^2 \\
 & \qquad 4x^3 - 4x^2 \\
 & \qquad\quad -3x^2 + 10x \\
 & \qquad\quad -3x^2 + 3x \\
 & \qquad\qquad 7x - 7 \\
 & \qquad\qquad 7x - 7 \\
 & \qquad\qquad 0
\end{array}
$$

Answer is $-5x^4 + 2x^3 + 4x^2 - 3x + 7$

(i)

$$
\begin{array}{r}
 & 2x^5 - 3x^4 + 2x^3 - 8x^2 + 4x + 6 \\
x - 4 \;\overline{\big)\; 2x^6 - 11x^5 + 14x^4 - 16x^3 + 36x^2 - 10x - 24} \\
 & 2x^6 - 8x^5 \\
 & \quad -3x^5 + 14x^4 \\
 & \quad -3x^5 + 12x^4 \\
 & \qquad 2x^4 - 16x^3 \\
 & \qquad 2x^4 - 8x^3 \\
 & \qquad\quad -8x^3 + 36x^2 \\
 & \qquad\quad -8x^3 + 32x^2 \\
 & \qquad\qquad 4x^2 - 10x \\
 & \qquad\qquad 4x^2 - 16x \\
 & \qquad\qquad\quad 6x - 24 \\
 & \qquad\qquad\quad 6x - 24 \\
 & \qquad\qquad\quad 0
\end{array}
$$

Answer is $2x^5 - 3x^4 + 2x^3 - 8x^2 + 4x + 6$

(j)

$$
\begin{array}{r|l}
x - 3 & \begin{array}{l} -x^5 + x^4 - x^3 + x^2 - 2x + 1 \\ \hline -x^6 + 4x^5 - 4x^4 + 4x^3 - 5x^2 + 7x - 3 \end{array} \\
 & \begin{array}{l}
-x^6 + 3x^5 \\
\quad x^5 - 4x^4 \\
\quad x^5 - 3x^4 \\
\qquad -x^4 + 4x^3 \\
\qquad -x^4 + 3x^3 \\
\qquad\quad x^3 - 5x^2 \\
\qquad\quad x^3 - 3x^2 \\
\qquad\qquad -2x^2 + 7x \\
\qquad\qquad -2x^2 + 6x \\
\qquad\qquad\quad x - 3 \\
\qquad\qquad\quad x - 3 \\
0
\end{array}
\end{array}
$$

Answer is $-x^5 + x^4 - x^3 + x^2 - 2x + 1$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise C, Question 1

Question:

Divide:

(a) $x^3 + x + 10$ by $(x + 2)$

(b) $2x^3 - 17x + 3$ by $(x + 3)$

(c) $-3x^3 + 50x - 8$ by $(x - 4)$

Solution:

(a)

$$\begin{array}{r|l} & x^2 - 2x + 5 \\ \hline x + 2 & x^3 + 0x^2 + x + 10 \\ & x^3 + 2x^2 \\ & \quad -2x^2 + x \\ & \quad -2x^2 - 4x \\ & \qquad 5x + 10 \\ & \qquad 5x + 10 \\ & \quad 0 \end{array}$$

Answer is $x^2 - 2x + 5$

(b)

$$\begin{array}{r|l} & 2x^2 - 6x + 1 \\ \hline x + 3 & 2x^3 + 0x^2 - 17x + 3 \\ & 2x^3 + 6x^2 \\ & \quad -6x^2 - 17x \\ & \quad -6x^2 - 18x \\ & \qquad x + 3 \\ & \qquad x + 3 \\ & \quad 0 \end{array}$$

Answer is $2x^2 - 6x + 1$

(c)

$$
\begin{array}{rl}
 & -3x^2 - 12x + 2 \\
x - 4 \;\overline{)} & -3x^3 + 0x^2 + 50x - 8 \\
 & -3x^3 + 12x^2 \\
 & \quad -12x^2 + 50x \\
 & \quad -12x^2 + 48x \\
 & \qquad\quad 2x - 8 \\
 & \qquad\quad 2x - 8 \\
 & \quad 0
\end{array}
$$

Answer is $-3x^2 - 12x + 2$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise C, Question 2

Question:

Divide:

(a) $x^3 + x^2 - 36$ by $(x - 3)$

(b) $2x^3 + 9x^2 + 25$ by $(x + 5)$

(c) $-3x^3 + 11x^2 - 20$ by $(x - 2)$

Solution:

(a)

$$
\begin{array}{r|l}
x - 3 & x^2 + 4x + 12 \\
\hline
 & x^3 + x^2 + 0x - 36 \\
 & x^3 - 3x^2 \\
 & \quad 4x^2 + 0x \\
 & \quad 4x^2 - 12x \\
 & \quad\quad 12x - 36 \\
 & \quad\quad 12x - 36 \\
 & \quad\quad 0
\end{array}
$$

Answer is $x^2 + 4x + 12$

(b)

$$
\begin{array}{r|l}
x + 5 & 2x^2 - x + 5 \\
\hline
 & 2x^3 + 9x^2 + 0x + 25 \\
 & 2x^3 + 10x^2 \\
 & \quad -x^2 + 0x \\
 & \quad -x^2 - 5x \\
 & \quad\quad 5x + 25 \\
 & \quad\quad 5x + 25 \\
 & \quad\quad 0
\end{array}
$$

Answer is $2x^2 - x + 5$

(c)

$$
\begin{array}{rl}
 & -3x^2 + 5x + 10 \\
x - 2 \;\big| & -3x^3 + 11x^2 + 0x - 20 \\
 & -3x^3 + 6x^2 \\
 & \quad 5x^2 + 0x \\
 & \quad 5x^2 - 10x \\
 & \qquad 10x - 20 \\
 & \qquad 10x - 20 \\
 & \quad 0
\end{array}
$$

Answer is $-3x^2 + 5x + 10$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise C, Question 3

Question:

Divide:

(a) $x^3 + 2x^2 - 5x - 10$ by $(x + 2)$

(b) $2x^3 - 6x^2 + 7x - 21$ by $(x - 3)$

(c) $-3x^3 + 21x^2 - 4x + 28$ by $(x - 7)$

Solution:

(a)
$$\begin{array}{r|l} & x^2 - 5 \\ \hline x + 2 & x^3 + 2x^2 - 5x - 10 \\ & x^3 + 2x^2 \\ & \quad 0 \quad - 5x - 10 \\ & \quad\quad - 5x - 10 \\ & 0 \end{array}$$

Answer is $x^2 - 5$

(b)
$$\begin{array}{r|l} & 2x^2 + 7 \\ \hline x - 3 & 2x^3 - 6x^2 + 7x - 21 \\ & 2x^3 - 6x^2 \\ & \quad 0 + \quad 7x - 21 \\ & \quad\quad 7x - 21 \\ & 0 \end{array}$$

Answer is $2x^2 + 7$

(c)
$$\begin{array}{r|l} & -3x^2 - 4 \\ \hline x - 7 & -3x^3 + 21x^2 - 4x + 28 \\ & -3x^3 + 21x^2 \\ & \quad 0 \quad - 4x + 28 \\ & \quad\quad - 4x + 28 \\ & 0 \end{array}$$

Answer is $-3x^2 - 4$

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Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise C, Question 4

Question:

Find the remainder when:

(a) $x^3 + 4x^2 - 3x + 2$ is divided by $(x + 5)$

(b) $3x^3 - 20x^2 + 10x + 5$ is divided by $(x - 6)$

(c) $-2x^3 + 3x^2 + 12x + 20$ is divided by $(x - 4)$

Solution:

(a)

$$\begin{array}{r|l} & x^2 - x + 2 \\ \hline x + 5 & x^3 + 4x^2 - 3x + 2 \\ & x^3 + 5x^2 \\ & \quad - x^2 - 3x \\ & \quad - x^2 - 5x \\ & \quad\quad 2x + 2 \\ & \quad\quad 2x + 10 \\ & -8 \end{array}$$

The remainder is -8.

(b)

$$\begin{array}{r|l} & 3x^2 - 2x - 2 \\ \hline x - 6 & 3x^3 - 20x^2 + 10x + 5 \\ & 3x^3 - 18x^2 \\ & \quad - 2x^2 + 10x \\ & \quad - 2x^2 + 12x \\ & \quad\quad - 2x + 5 \\ & \quad\quad - 2x + 12 \\ & -7 \end{array}$$

The remainder is -7.

(c)

$$
\begin{array}{rl}
 & -2x^2 - 5x - 8 \\
x - 4 \;\overline{\big)} & -2x^3 + 3x^2 + 12x + 20 \\
 & -2x^3 + 8x^2 \\
 & \quad -5x^2 + 12x \\
 & \quad -5x^2 + 20x \\
 & \qquad -8x + 20 \\
 & \qquad -8x + 32 \\
 & -12
\end{array}
$$

The remainder is -12.

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Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise C, Question 5

Question:

Show that when $3x^3 - 2x^2 + 4$ is divided by $(x - 1)$ the remainder is 5.

Solution:

$$
\begin{array}{r}
3x^2 + x + 1 \\
x - 1 \overline{\big)\ 3x^3 - 2x^2 + 0x + 4} \\
3x^3 - 3x^2 \\
\quad x^2 + 0x \\
\quad x^2 - x \\
\quad\quad x + 4 \\
\quad\quad x - 1 \\
\quad 5
\end{array}
$$

So the remainder is 5.

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Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise C, Question 6

Question:

Show that when $3x^4 - 8x^3 + 10x^2 - 3x - 25$ is divided by $(x + 1)$ the remainder is -1.

Solution:

$$
\begin{array}{rl}
 & 3x^3 - 11x^2 + 21x - 24 \\
x + 1 \;\overline{)} & 3x^4 - 8x^3 + 10x^2 - 3x - 25 \\
 & 3x^4 + 3x^3 \\
 & \quad -11x^3 + 10x^2 \\
 & \quad -11x^3 - 11x^2 \\
 & \qquad\quad 21x^2 - 3x \\
 & \qquad\quad 21x^2 + 21x \\
 & \qquad\qquad\quad -24x - 25 \\
 & \qquad\qquad\quad -24x - 24 \\
 & \qquad\qquad\quad -1
\end{array}
$$

So the remainder is -1.

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Algebra and functions

Exercise C, Question 7

Question:

Show that $(x + 4)$ is the factor of $5x^3 - 73x + 28$.

Solution:

$$
\begin{array}{r|l}
 & 5x^2 - 20x + 7 \\
\hline
x + 4 & 5x^3 + 0x^2 - 73x + 28 \\
 & 5x^3 + 20x^2 \\
 & \quad - 20x^2 - 73x \\
 & \quad - 20x^2 - 80x \\
 & \qquad\quad 7x + 28 \\
 & \qquad\quad 7x + 28 \\
 & \quad 0
\end{array}
$$

The remainder is 0, so $x + 4$ is a factor of $5x^3 - 73x + 28$.

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Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise C, Question 8

Question:

Simplify $\frac{3x^3 - 8x - 8}{x - 2}$.

Solution:

$$
\begin{array}{r}
3x^2 + 6x + 4 \\
x - 2 \overline{\big)\ 3x^3 + 0x^2 - 8x - 8} \\
3x^3 - 6x^2 \\
\hline
6x^2 - 8x \\
6x^2 - 12x \\
\hline
4x - 8 \\
4x - 8 \\
\hline
0
\end{array}
$$

So $\frac{3x^3 - 8x - 8}{x - 2} = 3x^2 + 6x + 4$.

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Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise C, Question 9

Question:

Divide $x^3 - 1$ by $(x - 1)$.

Solution:

$$
\begin{array}{r}
x^2 + x + 1 \\
x - 1 \overline{\big)\ x^3 + 0x^2 + 0x - 1} \\
x^3 - x^2 \\
\hline
x^2 + 0x \\
x^2 - x \\
\hline
x - 1 \\
x - 1 \\
\hline
0
\end{array}
$$

Answer is $x^2 + x + 1$.

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Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise C, Question 10

Question:

Divide $x^4 - 16$ by $(x + 2)$.

Solution:

$$
\begin{array}{r|l}
x + 2 & x^3 - 2x^2 + 4x - 8 \\
\hline
 & x^4 + 0x^3 + 0x^2 + 0x - 16 \\
 & x^4 + 2x^3 \\
 & \quad - 2x^3 + 0x^2 \\
 & \quad - 2x^3 - 4x^2 \\
 & \qquad\quad 4x^2 + 0x \\
 & \qquad\quad 4x^2 + 8x \\
 & \qquad\qquad\quad - 8x - 16 \\
 & \qquad\qquad\quad - 8x - 16 \\
 & \quad 0
\end{array}
$$

Answer is $x^3 - 2x^2 + 4x - 8$.

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Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise D, Question 1

Question:

Use the factor theorem to show:

(a) $(x - 1)$ is a factor of $4x^3 - 3x^2 - 1$

(b) $(x + 3)$ is a factor of $5x^4 - 45x^2 - 6x - 18$

(c) $(x - 4)$ is a factor of $-3x^3 + 13x^2 - 6x + 8$

Solution:

(a) $f(x) = 4x^3 - 3x^2 - 1$
$f(1) = 4(1)^3 - 3(1)^2 - 1 = 4 - 3 - 1 = 0$
So $(x - 1)$ is a factor of $4x^3 - 3x^2 - 1$

(b) $f(x) = 5x^4 - 45x^2 - 6x - 18$
$f(-3) = 5(-3)^4 - 45(-3)^2 - 6(-3) - 18$
$f(-3) = 5(81) - 45(9) + 18 - 18 = 405 - 405 = 0$
So $(x + 3)$ is a factor of $5x^4 - 45x^2 - 6x - 18$

(c) $f(x) = -3x^3 + 13x^2 - 6x + 8$
$f(4) = -3(4)^3 + 13(4)^2 - 6(4) + 8$
$f(4) = -192 + 208 - 24 + 8 = 0$
So $(x - 4)$ is a factor of $-3x^3 + 13x^2 - 6x + 8$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise D, Question 2

Question:

Show that $(x - 1)$ is a factor of $x^3 + 6x^2 + 5x - 12$ and hence factorise the expression completely.

Solution:

$f(x) = x^3 + 6x^2 + 5x - 12$
$f(1) = (1)^3 + 6(1)^2 + 5(1) - 12 = 1 + 6 + 5 - 12 = 0$
So $(x - 1)$ is a factor of $x^3 + 6x^2 + 5x - 12$

$$
\begin{array}{r|l}
x - 1 & x^2 + 7x + 12 \\
\hline
 & x^3 + 6x^2 + 5x - 12 \\
 & x^3 - x^2 \\
 & \quad 7x^2 + 5x \\
 & \quad 7x^2 - 7x \\
 & \quad\quad 12x - 12 \\
 & \quad\quad 12x - 12 \\
 & \quad 0
\end{array}
$$

Now $x^2 + 7x + 12 = (x + 3)(x + 4)$
So $x^3 + 6x^2 + 5x - 12 = (x - 1)(x + 3)(x + 4)$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise D, Question 3

Question:

Show that $(x + 1)$ is a factor of $x^3 + 3x^2 - 33x - 35$ and hence factorise the expression completely.

Solution:

$f(x) = x^3 + 3x^2 - 33x - 35$
$f(-1) = (-1)^3 + 3(-1)^2 - 33(-1) - 35 = -1 + 3 + 33 - 35 = 0$
So $(x + 1)$ is a factor of $x^3 + 3x^2 - 33x - 35$

$$
\begin{array}{r|l}
x + 1 & x^2 + 2x - 35 \\
\hline
 & x^3 + 3x^2 - 33x - 35 \\
 & x^3 + x^2 \\
 & \quad 2x^2 - 33x \\
 & \quad 2x^2 + 2x \\
 & \quad\quad -35x - 35 \\
 & \quad\quad -35x - 35 \\
 & \quad 0
\end{array}
$$

Now $x^2 + 2x - 35 = (x + 7)(x - 5)$
So $x^3 + 3x^2 - 33x - 35 = (x + 1)(x + 7)(x - 5)$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise D, Question 4

Question:

Show that $(x - 5)$ is a factor of $x^3 - 7x^2 + 2x + 40$ and hence factorise the expression completely.

Solution:

$f(x) = x^3 - 7x^2 + 2x + 40$
$f(5) = (5)^3 - 7(5)^2 + 2(5) + 40$
$f(5) = 125 - 175 + 10 + 40 = 0$
So $(x - 5)$ is a factor of $x^3 - 7x^2 + 2x + 40$

$$
\begin{array}{rl}
 & x^2 - 2x - 8 \\
x - 5 \;\overline{)} & x^3 - 7x^2 + 2x + 40 \\
 & x^3 - 5x^2 \\
 & \quad -2x^2 + 2x \\
 & \quad -2x^2 + 10x \\
 & \qquad -8x + 40 \\
 & \qquad -8x + 40 \\
 & \quad 0
\end{array}
$$

Now $x^2 - 2x - 8 = (x - 4)(x + 2)$
So $x^3 - 7x^2 + 2x + 40 = (x - 5)(x - 4)(x + 2)$.

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise D, Question 5

Question:

Show that $(x - 2)$ is a factor of $2x^3 + 3x^2 - 18x + 8$ and hence factorise the expression completely.

Solution:

$f(x) = 2x^3 + 3x^2 - 18x + 8$
$f(2) = 2(2)^3 + 3(2)^2 - 18(2) + 8 = 16 + 12 - 36 + 8 = 0$
So $(x - 2)$ is a factor of $2x^3 + 3x^2 - 18x + 8$

$$
\begin{array}{r|l}
x - 2 & 2x^2 + 7x - 4 \\
\cline{2-2}
 & 2x^3 + 3x^2 - 18x + 8 \\
 & 2x^3 - 4x^2 \\
 & \quad 7x^2 - 18x \\
 & \quad 7x^2 - 14x \\
 & \quad\quad - 4x + 8 \\
 & \quad\quad - 4x + 8 \\
 & \quad\quad 0
\end{array}
$$

Now $2x^2 + 7x - 4 = (2x - 1)(x + 4)$
So $2x^3 + 3x^2 - 18x + 8 = (x - 2)(2x - 1)(x + 4)$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise D, Question 6

Question:

Each of these expressions has a factor $(x \pm p)$. Find a value of p and hence factorise the expression completely.

(a) $x^3 - 10x^2 + 19x + 30$

(b) $x^3 + x^2 - 4x - 4$

(c) $x^3 - 4x^2 - 11x + 30$

Solution:

(a) $f(x) = x^3 - 10x^2 + 19x + 30$
$f(-1) = (-1)^3 - 10(-1)^2 + 19(-1) + 30 = -1 - 10 - 19 + 30 = 0$
So $(x + 1)$ is a factor.

$$
\begin{array}{r|l}
x + 1 & x^2 - 11x + 30 \\
\hline
 & x^3 - 10x^2 + 19x + 30 \\
 & x^3 + x^2 \\
 & \quad -11x^2 + 19x \\
 & \quad -11x^2 - 11x \\
 & \qquad\quad 30x + 30 \\
 & \qquad\quad 30x + 30 \\
 & \quad 0
\end{array}
$$

Now $x^2 - 11x + 30 = (x - 5)(x - 6)$
So $x^3 - 10x^2 + 19x + 30 = (x + 1)(x - 5)(x - 6)$.

(b) $f(x) = x^3 + x^2 - 4x - 4$
$f(-1) = (-1)^3 + (-1)^2 - 4(-1) - 4 = -1 + 1 + 4 - 4 = 0$
So $(x + 1)$ is a factor.

$$
\begin{array}{r|l}
x + 1 & x^2 - 4 \\
\hline
 & x^3 + x^2 - 4x - 4 \\
 & x^3 + x^2 \\
 & \quad 0 \quad -4x - 4 \\
 & \qquad -4x - 4 \\
 & \quad 0
\end{array}
$$

Now $x^2 - 4 = (x - 2)(x + 2)$
So $x^3 + x^2 - 4x - 4 = (x + 1)(x - 2)(x + 2)$

(c) $f(x) = x^3 - 4x^2 - 11x + 30$
$f(2) = (2)^3 - 4(2)^2 - 11(2) + 30 = 8 - 16 - 22 + 30 = 0$
So $(x - 2)$ is a factor.

$$
\begin{array}{r|l}
 & x^2 - 2x - 15 \\
\hline
x - 2 & x^3 - 4x^2 - 11x + 30 \\
 & x^3 - 2x^2 \\
 & \quad - 2x^2 - 11x \\
 & \quad - 2x^2 + 4x \\
 & \quad\quad - 15x + 30 \\
 & \quad\quad - 15x + 30 \\
 & \quad 0
\end{array}
$$

Now $x^2 - 2x - 15 = (x + 3)(x - 5)$
So $x^3 - 4x^2 - 11x + 30 = (x - 2)(x + 3)(x - 5)$.

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Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise D, Question 7

Question:

Factorise:

(a) $2x^3 + 5x^2 - 4x - 3$

(b) $2x^3 - 17x^2 + 38x - 15$

(c) $3x^3 + 8x^2 + 3x - 2$

(d) $6x^3 + 11x^2 - 3x - 2$

(e) $4x^3 - 12x^2 - 7x + 30$

Solution:

(a) $f(x) = 2x^3 + 5x^2 - 4x - 3$
$f(1) = 2(1)^3 + 5(1)^2 - 4(1) - 3 = 2 + 5 - 4 - 3 = 0$
So $(x - 1)$ is a factor.

$$
\begin{array}{r|l}
x - 1 & 2x^2 + 7x + 3 \\
\hline
 & 2x^3 + 5x^2 - 4x - 3 \\
 & 2x^3 - 2x^2 \\
 & \quad 7x^2 - 4x \\
 & \quad 7x^2 - 7x \\
 & \quad\quad 3x - 3 \\
 & \quad\quad 3x - 3 \\
 & \quad\quad 0
\end{array}
$$

Now $2x^2 + 7x + 3 = (2x + 1)(x + 3)$
So $2x^3 + 5x^2 - 4x - 3 = (x - 1)(2x + 1)(x + 3)$.

(b) $f(x) = 2x^3 - 17x^2 + 38x - 15$
$f(3) = 2(3)^3 - 17(3)^2 + 38(3) - 15 = 54 - 153 + 114 - 15 = 0$
So $(x - 3)$ is a factor.

$$
\begin{array}{r|l}
x - 3 & 2x^2 - 11x + 5 \\
\hline
 & 2x^3 - 17x^2 + 38x - 15 \\
 & 2x^3 - 6x^2 \\
 & \quad -11x^2 + 38x \\
 & \quad -11x^2 + 33x \\
 & \quad\quad 5x - 15 \\
 & \quad\quad 5x - 15 \\
 & \quad\quad 0
\end{array}
$$

Now $2x^2 - 11x + 5 = (2x - 1)(x - 5)$
So $2x^3 - 17x^2 + 38x - 15 = (x - 3)(2x - 1)(x - 5)$.

(c) $f(x) = 3x^3 + 8x^2 + 3x - 2$
$f(-1) = 3(-1)^3 + 8(-1)^2 + 3(-1) - 2 = -3 + 8 - 3 - 2 = 0$
So $(x + 1)$ is a factor.

$$
\begin{array}{r|l}
x + 1 & 3x^2 + 5x - 2 \\
\cline{2-2}
 & 3x^3 + 8x^2 + 3x - 2 \\
 & 3x^3 + 3x^2 \\
 & \quad 5x^2 + 3x \\
 & \quad 5x^2 + 5x \\
 & \quad\quad -2x - 2 \\
 & \quad\quad -2x - 2 \\
 & \quad\quad 0
\end{array}
$$

Now $3x^2 + 5x - 2 = (3x - 1)(x + 2)$
So $3x^3 + 8x^2 + 3x - 2 = (x + 1)(3x - 1)(x + 2)$.

(d) $f(x) = 6x^3 + 11x^2 - 3x - 2$
$f(-2) = 6(-2)^3 + 11(-2)^2 - 3(-2) - 2 = -48 + 44 + 6 - 2 = 0$
So $(x + 2)$ is a factor.

$$
\begin{array}{r|l}
x + 2 & 6x^2 - x - 1 \\
\cline{2-2}
 & 6x^3 + 11x^2 - 3x - 2 \\
 & 6x^3 + 12x^2 \\
 & \quad -x^2 - 3x \\
 & \quad -x^2 - 2x \\
 & \quad\quad -x - 2 \\
 & \quad\quad -x - 2 \\
 & \quad\quad 0
\end{array}
$$

Now $6x^2 - x - 1 = (3x + 1)(2x - 1)$
So $6x^3 + 11x^2 - 3x - 2 = (x + 2)(3x + 1)(2x - 1)$.

(e) $f(x) = 4x^3 - 12x^2 - 7x + 30$
$f(2) = 4(2)^3 - 12(2)^2 - 7(2) + 30 = 32 - 48 - 14 + 30 = 0$
So $(x - 2)$ is a factor.

$$
\begin{array}{r|l}
x - 2 & 4x^2 - 4x - 15 \\
\cline{2-2}
 & 4x^3 - 12x^2 - 7x + 30 \\
 & 4x^3 - 8x^2 \\
 & \quad -4x^2 - 7x \\
 & \quad -4x^2 + 8x \\
 & \quad\quad -15x + 30 \\
 & \quad\quad -15x + 30 \\
 & \quad\quad 0
\end{array}
$$

Now $4x^2 - 4x - 15 = (2x + 3)(2x - 5)$
So $4x^3 - 12x^2 - 7x + 30 = (x - 2)(2x + 3)(2x - 5)$.

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise D, Question 8

Question:

Given that $(x - 1)$ is a factor of $5x^3 - 9x^2 + 2x + a$ find the value of a.

Solution:

$f(x) = 5x^3 - 9x^2 + 2x + a$
$f(1) = 0$
So $5(1)^3 - 9(1)^2 + 2(1) + a = 0$
$5 - 9 + 2 + a = 0$
$a = 2$

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Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise D, Question 9

Question:

Given that $(x + 3)$ is a factor of $6x^3 - bx^2 + 18$ find the value of b.

Solution:

$f(x) = 6x^3 - bx^2 + 18$
$f(-3) = 0$
So $6(-3)^3 - b(-3)^2 + 18 = 0$
$-162 - 9b + 18 = 0$
$9b = -144$
$b = -16$

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Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise D, Question 10

Question:

Given that $(x - 1)$ and $(x + 1)$ are factors of $px^3 + qx^2 - 3x - 7$ find the value of p and q.

Solution:

$f(x) = px^3 + qx^2 - 3x - 7$

① $f(1) = 0$

$p(1)^3 + q(1)^2 - 3(1) - 7 = 0$

$p + q - 3 - 7 = 0$

$p + q = 10$

② $f(-1) = 0$

$p(-1)^3 + q(-1)^2 - 3(-1) - 7 = 0$

$-p + q + 3 - 7 = 0$

$-p + q = 4$

Solve simultaneously:

$$\begin{aligned} p + q &= 10 \\ -p + q &= 4 \\ 2q &= 14 \\ q &= 7 \end{aligned}$$

$p + q = 10$, so $p = 3$.

Answer is $p = 3$, $q = 7$.

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise E, Question 1

Question:

Find the remainder when:

(a) $4x^3 - 5x^2 + 7x + 1$ is divided by $(x - 2)$

(b) $2x^5 - 32x^3 + x - 10$ is divided by $(x - 4)$

(c) $-2x^3 + 6x^2 + 5x - 3$ is divided by $(x + 1)$

(d) $7x^3 + 6x^2 - 45x + 1$ is divided by $(x + 3)$

(e) $4x^4 - 4x^2 + 8x - 1$ is divided by $(2x - 1)$

(f) $243x^4 - 27x^3 - 3x + 7$ is divided by $(3x - 1)$

(g) $64x^3 + 32x^2 + 16x + 9$ is divided by $(4x + 1)$

(h) $81x^3 - 81x^2 + 9x + 6$ is divided by $(3x - 2)$

(i) $243x^6 - 780x^2 + 6$ is divided by $(3x + 4)$

(j) $125x^4 + 5x^3 - 9x$ is divided by $(5x + 3)$

Solution:

(a) $f(x) = 4x^3 - 5x^2 + 7x + 1$
$f(2) = 4(2)^3 - 5(2)^2 + 7(2) + 1$
$f(2) = 32 - 20 + 14 + 1 = 27$
Remainder is 27.

(b) $f(x) = 2x^5 - 32x^3 + x - 10$
$f(4) = 2(4)^5 - 32(4)^3 + (4) - 10$
$f(4) = 2048 - 2048 + 4 - 10 = -6$
Remainder is -6.

(c) $f(x) = -2x^3 + 6x^2 + 5x - 3$
$f(-1) = -2(-1)^3 + 6(-1)^2 + 5(-1) - 3$
$f(-1) = 2 + 6 - 5 - 3 = 0$
Remainder is 0.

(d) $f(x) = 7x^3 + 6x^2 - 45x + 1$
$f(-3) = 7(-3)^3 + 6(-3)^2 - 45(-3) + 1$
$f(-3) = -189 + 54 + 135 + 1 = 1$
Remainder is 1.

(e) $f(x) = 4x^4 - 4x^2 + 8x - 1$
$f\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^4 - 4\left(\frac{1}{2}\right)^2 + 8\left(\frac{1}{2}\right) - 1$

$f\left(\frac{1}{2}\right) = \frac{1}{4} - 1 + 4 - 1 = 2\frac{1}{4}$

Remainder is $2\frac{1}{4}$.

(f) $f(x) = 243x^4 - 27x^3 - 3x + 7$

$f\left(\frac{1}{3}\right) = 243\left(\frac{1}{3}\right)^4 - 27\left(\frac{1}{3}\right)^3 - 3\left(\frac{1}{3}\right) + 7$

$f\left(\frac{1}{3}\right) = 3 - 1 - 1 + 7 = 8$

Remainder is 8.

(g) $f(x) = 64x^3 + 32x^2 - 16x + 9$

$f\left(-\frac{1}{4}\right) = 64\left(-\frac{1}{4}\right)^3 + 32\left(-\frac{1}{4}\right)^2 - 16\left(-\frac{1}{4}\right) + 9$

$f\left(-\frac{1}{4}\right) = -1 + 2 + 4 + 9 = 14$

Remainder is 14.

(h) $f(x) = 81x^3 - 81x^2 + 9x + 6$

$f\left(\frac{2}{3}\right) = 81\left(\frac{2}{3}\right)^3 - 81\left(\frac{2}{3}\right)^2 + 9\left(\frac{2}{3}\right) + 6$

$f\left(\frac{2}{3}\right) = 24 - 36 + 6 + 6 = 0$

Remainder is 0.

(i) $f(x) = 243x^6 - 780x^2 + 6$

$f\left(-\frac{4}{3}\right) = 243\left(-\frac{4}{3}\right)^6 - 780\left(-\frac{4}{3}\right)^2 + 6$

$f\left(-\frac{4}{3}\right) = \frac{4096}{3} - \frac{4160}{3} + 6 = -\frac{64}{3} + 6 = -21\frac{1}{3} + 6 = -15\frac{1}{3}$

Remainder is $-15\frac{1}{3}$.

(j) $f(x) = 125x^4 + 5x^3 - 9x$

$f\left(-\frac{3}{5}\right) = 125\left(-\frac{3}{5}\right)^4 + 5\left(-\frac{3}{5}\right)^3 - 9\left(-\frac{3}{5}\right)$

$f\left(-\frac{3}{5}\right) = \frac{405}{25} - \frac{27}{25} + \frac{27}{5} = \frac{378}{25} + \frac{135}{25} = \frac{513}{25} = 20\frac{13}{25}$

Remainder is $20\frac{13}{25}\left(= 20.52\right)$.

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise E, Question 2

Question:

When $2x^3 - 3x^2 - 2x + a$ is divided by $(x - 1)$ the remainder is -4. Find the value of a.

Solution:

$f(x) = 2x^3 - 3x^2 - 2x + a$
$f(1) = -4$
So $2(1)^3 - 3(1)^2 - 2(1) + a = -4$
$2 - 3 - 2 + a = -4$
$a = -1$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise E, Question 3

Question:

When $-3x^3 + 4x^2 + bx + 6$ is divided by $(x + 2)$ the remainder is 10. Find the value of b.

Solution:

$f(x) = -3x^3 + 4x^2 + bx + 6$
$f(-2) = 10$
So $-3(-2)^3 + 4(-2)^2 + b(-2) + 6 = 10$
$24 + 16 - 2b + 6 = 10$
$2b = 36$
$b = 18$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise E, Question 4

Question:

When $16x^3 - 32x^2 + cx - 8$ is divided by by $(2x - 1)$ the remainder is 1. Find the value of c.

Solution:

$f(x) = 16x^3 - 32x^2 + cx - 8$

$f\left(\frac{1}{2}\right) = 1$

So $16\left(\frac{1}{2}\right)^3 - 32\left(\frac{1}{2}\right)^2 + c\left(\frac{1}{2}\right) - 8 = 1$

$2 - 8 + \frac{1}{2}c - 8 = 1$

$\frac{1}{2}c = 15$

$c = 30$

Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Algebra and functions
Exercise E, Question 5

Question:

Show that $(x - 3)$ is a factor of $x^6 - 36x^3 + 243$.

Solution:

$f(x) = x^6 - 36x^3 + 243$
$f(3) = (3)^6 - 36(3)^3 + 243$
$f(3) = 729 - 972 + 243 = 0$
Remainder is 0, so $(x - 3)$ is a factor.

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise E, Question 6

Question:

Show that $(2x - 1)$ is a factor of $2x^3 + 17x^2 + 31x - 20$.

Solution:

$f(x) = 2x^3 + 17x^2 + 31x - 20$

$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + 17\left(\frac{1}{2}\right)^2 + 31\left(\frac{1}{2}\right) - 20$

$f\left(\frac{1}{2}\right) = \frac{1}{4} + \frac{17}{4} + \frac{31}{2} - 20 = \frac{1 + 17 + 62 - 80}{4} = 0$

Remainder is 0, so $(2x - 1)$ is a factor.

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise E, Question 7

Question:

f (x) = x^2 + 3x + q . Given f (2) = 3 , find f (− 2) .

Solution:

f (x) = x^2 + 3x + q
Given f (2) = 3 .
So (2) ^{2} + 3 (2) + q = 3
4 + 6 + q = 3
q = − 7
f (x) = x^2 + 3x − 7
f (− 2) = (− 2) ^{2} + 3 (− 2) − 7
f (− 2) = 4 − 6 − 7 = − 9
Answer is − 9.

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise E, Question 8

Question:

$g(x) = x^3 + ax^2 + 3x + 6$. Given $g(-1) = 2$, find the remainder when $g(x)$ is divided by $(3x - 2)$.

Solution:

$g(x) = x^3 + ax^2 + 3x + 6$

Given $g(-1) = 2$.

So $(-1)^3 + a(-1)^2 + 3(-1) + 6 = 2$

$-1 + a - 3 + 6 = 2$

$a = 0$

$g(x) = x^3 + 3x + 6$

$g\left(\frac{2}{3}\right) = \left(\frac{2}{3}\right)^3 + 3\left(\frac{2}{3}\right) + 6$

$g\left(\frac{2}{3}\right) = \frac{8}{27} + 2 + 6 = 8\frac{8}{27}$

Answer is $8\frac{8}{27}$.

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise E, Question 9

Question:

The expression $2x^3 - x^2 + ax + b$ gives a remainder 14 when divided by $(x - 2)$ and a remainder -86 when divided by $(x + 3)$. Find the value of a and b.

Solution:

$f(x) = 2x^3 - x^2 + ax + b$

① $f(2) = 14$

So $2(2)^3 - (2)^2 + a(2) + b = 14$

$16 - 4 + 2a + b = 14$

$2a + b = 2$

② $f(-3) = -86$

So $2(-3)^3 - (-3)^2 + a(-3) + b = -86$

$-54 - 9 - 3a + b = -86$

$-3a + b = -23$

Solve simultaneously:

$$\begin{aligned} 2a + b &= 2 \\ -3a + b &= -23 \\ 5a &= 25 \end{aligned}$$

$a = 5$

$2a + b = 2$

Substitute $a = 5$:

$2(5) + b = 2$

$10 + b = 2$

$b = -8$

Check $a = 5$, $b = -8$ by substitution:

$-3a + b = -3(5) + (-8) = -15 - 8 = -23$ ✓

Answer is $a = 5$, $b = -8$.

Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Algebra and functions
Exercise E, Question 10

Question:

The expression $3x^3 + 2x^2 - px + q$ is divisible by $(x - 1)$ but leaves a remainder of 10 when divided by $(x + 1)$. Find the value of a and b.

Solution:

$f(x) = 3x^3 + 2x^2 - px + q$

① $f(1) = 0$

So $3(1)^3 + 2(1)^2 - p(1) + q = 0$

$3 + 2 - p + q = 0$

$-p + q = -5$

② $f(-1) = 10$

So $3(-1)^3 + 2(-1)^2 - p(-1) + q = 0$

$-3 + 2 + p + q = 10$

$p + q = 11$

Solve simultaneously:

$$-p + q = -5$$
$$p + q = 11$$
$$2q = 6$$

$q = 3$

Substitute $q = 3$:

$p + q = 11$

$p + 3 = 11$

$p = 8$

Check: $-p + q = -8 + 3 = -5$ ✓

Answer is $p = 8$, $q = 3$.

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise F, Question 1

Question:

Simplify these fractions as far as possible:

(a) $\frac{3x^4 - 21x}{3x}$

(b) $\frac{x^2 - 2x - 24}{x^2 - 7x + 6}$

(c) $\frac{2x^2 + 7x - 4}{2x^2 + 9x + 4}$

Solution:

(a) $\frac{3x^4 - 21x}{3x} = \frac{3x^4}{3x} - \frac{21x}{3x} = x^3 - 7$

(b) $\frac{x^2 - 2x - 24}{x^2 - 7x + 6}$

$= \frac{(x-6)(x+4)}{(x-6)(x-1)}$

$= \frac{x + 4}{x - 1}$

(c) $\frac{2x^2 + 7x - 4}{2x^2 + 9x + 4}$

$= \frac{(2x-1)(x+4)}{(2x+1)(x+4)}$

$= \frac{2x - 1}{2x + 1}$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise F, Question 2

Question:

Divide $3x^3 + 12x^2 + 5x + 20$ by $(x + 4)$.

Solution:

$$
\begin{array}{rl}
 & 3x^2 + 5 \\
x + 4 \;\overline{\big)} & 3x^3 + 12x^2 + 5x + 20 \\
 & 3x^3 + 12x^2 \\
 & \quad 0 + 5x + 20 \\
 & \quad\;\; 5x + 20 \\
 & \quad\;\; 0
\end{array}
$$

Answer is $3x^2 + 5$.

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise F, Question 3

Question:

Simplify $\frac{2x^3 + 3x + 5}{x + 1}$.

Solution:

$$
\begin{array}{r}
2x^2 - 2x + 5 \\
x + 1 \overline{\big)\ 2x^3 + 0x^2 + 3x + 5} \\
2x^3 + 2x^2 \\
\quad - 2x^2 + 3x \\
\quad - 2x^2 - 2x \\
\quad\quad 5x + 5 \\
\quad\quad 5x + 5 \\
\quad 0
\end{array}
$$

So $\frac{2x^3 + 3x + 5}{x + 1} = 2x^2 - 2x + 5$.

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise F, Question 4

Question:

Show that $(x - 3)$ is a factor of $2x^3 - 2x^2 - 17x + 15$. Hence express $2x^3 - 2x^2 - 17x + 15$ in the form $\begin{pmatrix} x - 3 \end{pmatrix}$ $\begin{pmatrix} Ax^2 + Bx + C \end{pmatrix}$, where the values A, B and C are to be found.

Solution:

$f(x) = 2x^3 - 2x^2 - 17x + 15$
$f(3) = 2(3)^3 - 2(3)^2 - 17(3) + 15$
$f(3) = 54 - 18 - 51 + 15 = 0$
So $(x - 3)$ is a factor.

$$
\begin{array}{rl}
 & 2x^2 + 4x - 5 \\
x - 3 \;\overline{)} & 2x^3 - 2x^2 - 17x + 15 \\
 & 2x^3 - 6x^2 \\
 & \quad 4x^2 - 17x \\
 & \quad 4x^2 - 12x \\
 & \quad\quad - 5x + 15 \\
 & \quad\quad - 5x + 15 \\
 & \quad\quad 0
\end{array}
$$

So $2x^3 - 2x^2 - 17x + 15 = (x - 3)(2x^2 + 4x - 5)$.
So $A = 2$, $B = 4$, $C = -5$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise F, Question 5

Question:

Show that $(x - 2)$ is a factor of $x^3 + 4x^2 - 3x - 18$. Hence express $x^3 + 4x^2 - 3x - 18$ in the form $(x - 2)(px + q)^2$, where the values p and q are to be found.

Solution:

$f(x) = x^3 + 4x^2 - 3x - 18$
$f(2) = (2)^3 + 4(2)^2 - 3(2) - 18$
$f(2) = 8 + 16 - 6 - 18 = 0$
So $(x - 2)$ is a factor.

$$
\begin{array}{rl}
 & x^2 + 6x + 9 \\
x - 2 \;\big| & x^3 + 4x^2 - 3x - 18 \\
 & x^3 - 2x^2 \\
 & \quad 6x^2 - 3x \\
 & \quad 6x^2 - 12x \\
 & \quad\quad 9x - 18 \\
 & \quad\quad 9x - 18 \\
 & \quad 0
\end{array}
$$

Now $x^2 + 6x + 9 = (x + 3)(x + 3) = (x + 3)^2$
So $x^3 + 4x^2 - 3x - 18 = (x - 2)(x + 3)^2$.
So $p = 1$, $q = 3$.

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise F, Question 6

Question:

Factorise completely $2x^3 + 3x^2 - 18x + 8$.

Solution:

$f(x) = 2x^3 + 3x^2 - 18x + 8$
$f(2) = 2(2)^3 + 3(2)^2 - 18(2) + 8$
$f(2) = 16 + 12 - 36 + 8 = 0$
So $(x - 2)$ is a factor.

$$
\begin{array}{r|l}
x - 2 & 2x^2 + 7x - 4 \\
\cline{2-2}
 & 2x^3 + 3x^2 - 18x + 8 \\
 & 2x^3 - 4x^2 \\
 & \quad 7x^2 - 18x \\
 & \quad 7x^2 - 14x \\
 & \quad\quad - 4x + 8 \\
 & \quad\quad - 4x + 8 \\
 & \quad\quad 0
\end{array}
$$

Now $2x^2 + 7x - 4 = (2x - 1)(x + 4)$
So $2x^3 + 3x^2 - 18x + 8 = (x - 2)(2x - 1)(x + 4)$.

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise F, Question 7

Question:

Find the value of k if $(x - 2)$ is a factor of $x^3 - 3x^2 + kx - 10$.

Solution:

$f(x) = x^3 - 3x^2 + kx - 10$
$f(2) = 0$
So $(2)^3 - 3(2)^2 + k(2) - 10 = 0$
$8 - 12 + 2k - 10 = 0$
$2k = 14$
$k = 7$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise F, Question 8

Question:

Find the remainder when $16x^5 - 20x^4 + 8$ is divided by $(2x - 1)$.

Solution:

$f(x) = 16x^5 - 20x^4 + 8$

$f\left(\frac{1}{2}\right) = 16\left(\frac{1}{2}\right)^5 - 20\left(\frac{1}{2}\right)^4 + 8$

$f\left(\frac{1}{2}\right) = \frac{1}{2} - \frac{5}{4} + 8 = 7\frac{1}{4}$

Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Algebra and functions
Exercise F, Question 9

Question:

f (x) = $2x^2 + px + q$. Given that f (− 3) = 0 , and f (4) = 2 :

(a) find the value of p and q

(b) factorise f (x)

Solution:

(a) f (x) = $2x^2 + px + q$
①f (− 3) = 0
So $2 (-3)^2 + p (-3) + q = 0$
$18 - 3p + q = 0$
$3p - q = 18$
②f (4) = 21
So $2 (4)^2 + p (4) + q = 21$
$4p + q = -11$
Solving simultaneously:

$$3p - q = 18$$
$$4p + q = -11$$
$$7p = 7$$

$p = 1$

Substitute $p = 1$ into $4p + q = -11$:
$4 (1) + q = -11$
$q = -15$

Check: $3p - q = 3 (1) - (-15) = 3 + 15 = 18$ ✓
So $p = 1$, $q = -15$

(b) f (x) = $2x^2 + x - 15 = (2x - 5) (x + 3)$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise F, Question 10

Question:

h (x) = $x^3 + 4x^2 + rx + s$. Given h (-1) = 0 , and h (2) = 30 :

(a) find the value of r and s

(b) find the remainder when h (x) is divided by ($3x - 1$)

Solution:

(a) h (x) = $x^3 + 4x^2 + rx + s$
① h (-1) = 0
So $(-1)^3 + 4(-1)^2 + r(-1) + s = 0$
$-1 + 4 - r + s = 0$
$-r + s = -3$
② h (2) = 30
So $(2)^3 + 4(2)^2 + r(2) + s = 30$
$8 + 16 + 2r + s = 30$
$2r + s = 6$
Solving simultaneously:

$$\begin{aligned} 2r + s &= 6 \\ -r + s &= -3 \\ 3r &= 9 \end{aligned}$$

$r = 3$

Substitute $r = 3$ into $-r + s = -3$:
$-3 + s = -3$
$s = 0$

Check: $2r + s = 2(3) + (0) = 6$ ✓
So $r = 3$, $s = 0$

(b) h (x) = $x^3 + 4x^2 + 3x$

$$h\left(\frac{1}{3}\right) = \left(\frac{1}{3}\right)^3 + 4\left(\frac{1}{3}\right)^2 + 3\left(\frac{1}{3}\right)$$

$$h\left(\frac{1}{3}\right) = \frac{1}{27} + \frac{4}{9} + 1 = 1\frac{13}{27}$$

Remainder is $1\frac{13}{27}$.

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise F, Question 11

Question:

$g(x) = 2x^3 + 9x^2 - 6x - 5$.

(a) Factorise $g(x)$

(b) Solve $g(x) = 0$

Solution:

(a) $g(x) = 2x^3 + 9x^2 - 6x - 5$
$g(1) = 2(1)^3 + 9(1)^2 - 6(1) - 5$
$g(1) = 2 + 9 - 6 - 5 = 0$
So $(x - 1)$ is a factor.

$$
\begin{array}{r|l}
x - 1 & 2x^2 + 11x + 5 \\
\hline
 & 2x^3 + 9x^2 - 6x - 5 \\
 & 2x^3 - 2x^2 \\
 & \quad 11x^2 - 6x \\
 & \quad 11x^2 - 11x \\
 & \quad\quad 5x - 5 \\
 & \quad\quad 5x - 5 \\
 & \quad\quad 0
\end{array}
$$

Now $2x^2 + 11x + 5 = (2x + 1)(x + 5)$
So $g(x) = (x - 1)(2x + 1)(x + 5)$

(b) $g(x) = 0$
$(x - 1)(2x + 1)(x + 5) = 0$
So $x = 1$, $x = -\frac{1}{2}$, $x = -5$.

Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Algebra and functions
Exercise F, Question 12

Question:

The remainder obtained when $x^3 - 5x^2 + px + 6$ is divided by $(x + 2)$ is equal to the remainder obtained when the same expression is divided by $(x - 3)$.
Find the value of p.

Solution:

$g(x) = x^3 - 5x^2 + px + 6$
① $g(-2) = R$
So $(-2)^3 - 5(-2)^2 + p(-2) + 6 = R$
$-8 - 20 - 2p + 6 = R$
$-2p - 22 = R$
② $g(3) = R$
So $(3)^3 - 5(3)^2 + p(3) + 6 = R$
$27 - 45 + 3p + 6 = R$
$3p - 12 = R$
Solving simultaneously:
$-2p - 22 = 3p - 12$
$-5p = 10$
$p = -2$

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Edexcel Modular Mathematics for AS and A-Level

Algebra and functions
Exercise F, Question 13

Question:

The remainder obtained when $x^3 + dx^2 - 5x + 6$ is divided by $(x - 1)$ is twice the remainder obtained when the same expression is divided by $(x + 1)$.
Find the value of d.

Solution:

$f(x) = x^3 + dx^2 - 5x + 6$
Let $f(-1) = R$
So $(-1)^3 + d(-1)^2 - 5(-1) + 6 = R$
$-1 + d + 5 + 6 = R$
$d + 10 = R$
Now $f(1) = 2R$
So $(1)^3 + d(1)^2 - 5(1) + 6 = 2R$
$1 + d - 5 + 6 = 2R$
$d + 2 = 2R$
Solving simultaneously:
$d + 2 = 2(d + 10)$
$d + 2 = 2d + 20$
$2 = d + 20$
$d = -18$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise F, Question 14

Question:

(a) Show that $(x - 2)$ is a factor of $f(x) = x^3 + x^2 - 5x - 2$.

(b) Hence, or otherwise, find the exact solutions of the equation $f(x) = 0$.

[E]

Solution:

(a) $f(x) = x^3 + x^2 - 5x - 2$
$f(2) = (2)^3 + (2)^2 - 5(2) - 2$
$f(2) = 8 + 4 - 10 - 2 = 0$
So $(x - 2)$ is a factor.

(b)
$$\begin{array}{r|l} & x^2 + 3x + 1 \\ x - 2 & \overline{\ x^3 + x^2 - 5x - 2} \\ & x^3 - 2x^2 \\ & \quad 3x^2 - 5x \\ & \quad 3x^2 - 6x \\ & \qquad\quad x - 2 \\ & \qquad\quad x - 2 \\ & \qquad\quad 0 \end{array}$$

So $f(x) = (x - 2)(x^2 + 3x + 1)$
Now $f(x) = 0$ when $x = 2$
and $x^2 + 3x + 1 = 0$

i.e. $x = \dfrac{-3 \pm \sqrt{(3)^2 - 4(1)(1)}}{2(1)}$ $\left(x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right)$

$\Rightarrow \quad x = \dfrac{-3 \pm \sqrt{5}}{2}$

So $x = 2$, $x = \dfrac{-3 + \sqrt{5}}{2}$, $x = \dfrac{-3 - \sqrt{5}}{2}$

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Algebra and functions

Exercise F, Question 15

Question:

Given that -1 is a root of the equation $2x^3 - 5x^2 - 4x + 3$, find the two positive roots.

[E]

Solution:

$$
\begin{array}{r|l}
 & 2x^2 - 7x + 3 \\
\hline
x + 1 & 2x^3 - 5x^2 - 4x + 3 \\
 & 2x^3 + 2x^2 \\
 & \quad -7x^2 - 4x \\
 & \quad -7x^2 - 7x \\
 & \qquad\quad 3x + 3 \\
 & \qquad\quad 3x + 3 \\
 & \quad 0
\end{array}
$$

Now $2x^2 - 7x + 3 = (2x - 1)(x - 3)$
So $2x^3 - 5x^2 - 4x + 3 = (x + 1)(2x - 1)(x - 3)$.

The roots are -1, $\frac{1}{2}$ and 3.

The positive roots are $x = \frac{1}{2}$ and $x = 3$.